Ibrahim Intabli

American University of Beirut

MATH 202

Differential Equations

Spring 2010

Final Exam

Useful formulas are given on the backside of the paper

PUT YOUR SECTION'S NUMBER ON YOUR BOOKLET PLEASE:

Dr LYZZAIK: **1 (F - 1pm)** **2 (F - 12pm)** **3 (F - 10am)** **4 (F - 9am)**

Dr KOBEISSI: **13 (T - 11am)** **14 (T - 12:30pm)** **15 (T - 2pm)** **16 (T - 9:30am)**

Exercise 1 *(10 points)* Given that $y = \sin x$ is a solution of

$$y^{(4)} + 2y''' + 11y'' + 2y' + 10y = 0$$

find the general solution of the DE

Exercise 2 *(10 points)* Solve the DE: $x^2y'' + xy' - y = \ln x$ on the interval $(0, +\infty)$

Exercise 3 *(15 points)* Find two power series solutions of the equation

$$y'' - 2xy' + y = 0$$

about the **ordinary point** $x = 0$

(give the radius of convergence of the series)

Exercise 4 *(20 points)* Evaluate:

(i) $\mathcal{L}\{\cos(2t)\mathcal{U}(t-\pi)\}$ (ii) $\mathcal{L}^{-1}\left\{\dfrac{e^{-s}}{s(s+1)}\right\}$ (iii) $\mathcal{L}\left\{t\displaystyle\int_0^t \sin\tau\, d\tau\right\}$

(iv)) $\mathcal{L}^{-1}\left\{\dfrac{1}{s^2+3s-4}\right\}$

Exercise 5 *(15 points)* Use Frobenius method to find **ONE** series solution *(corresponding to the larger root)* of the equation

$$x(x-1)y'' + 3y' - 2y = 0$$

about the **regular singular point** $x = 0$.

Write the solution in concise form.

Exercise 6 *(10 points)* Show that $y = \sqrt{x}J_{3/2}(x)$ is a solution of the differential equation

$$x^2y'' + \left(x^2 - 2\right)y = 0$$

Exercise 7 *(10 points)* Use the Laplace transform to solve the **IVP**

$$\begin{cases} y'' + 4y' + 13y = \delta(t-\pi) + \delta(t-3\pi) \\ y(0) = 1\,, \quad y'(0) = 0 \end{cases}$$

Exercise 8 *(10 points)* Find the general solution of the given system

$$\begin{cases} \dfrac{dx}{dt} = 3x - y - z \\ \dfrac{dy}{dt} = x + y - z \\ \dfrac{dz}{dt} = x - y + z \end{cases}$$

good luck

Bessel Equation:

The solution of the Bessel equation

$$x^2y'' + xy' + (x^2 - \nu^2)y = 0$$

is $y = c_1 J_\nu(x) + c_2 Y_\nu(x) \quad \text{for } x > 0$

The Bessel function $J_\nu = \sum_{n=0}^{+\infty} \frac{(-1)^n}{n!\, \Gamma(1+n+\nu)} \left(\frac{x}{2}\right)^{2n+\nu}$

Laplace Transform:

- $\mathcal{L}\{f(t)\} = \int_0^{+\infty} e^{-st} f(t)dt$
- $\mathcal{L}\{f^{(n)}t)\} = s^n F(s) - s^{n-1}f(0) - s^{n-2}f'(0) - \ldots - f^{(n-1)}(0)$
- $\mathcal{L}\{e^{at}f(t)\} = F(s-a)$
- $\mathcal{L}\{f(t-a)\mathcal{U}(t-a)\} = e^{-as}F(s)$
- $\mathcal{L}\{\mathcal{U}(t-a)\} = \frac{e^{-as}}{s}$
- $\mathcal{L}\{g(t)\mathcal{U}(t-a)\} = e^{-as}\mathcal{L}\{g(t+a)\}$
- $\mathcal{L}\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n} F(s)$
- $f * g = \int_0^t f(\tau)g(t-\tau)d\tau$
- $\mathcal{L}\{f * g\} = \mathcal{L}\{f\}\mathcal{L}\{g\}$

$\Gamma(n+1)$

- $\mathcal{L}\{\delta(t-t_0)\} = e^{-st_0}$